OCR
RECOGNISING ACHIEVEMENT

**ADVANCED SUBSIDIARY GCE**

# MATHEMATICS

Further Pure Mathematics 1

**4725**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**
- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**
- Scientific or graphical calculator

**Friday 11 June 2010**
**Morning**

**Duration:** 1 hour 30 minutes


**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

**1** Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. **[5]**

**2** The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$. Find

**(i)** **AB**, **[2]**

**(ii)** **BA** $- 4\mathbf{C}$. **[4]**

**3** Find $\sum_{r=1}^{n}(2r-1)^2$, expressing your answer in a fully factorised form. **[6]**

**4** The complex numbers $a$ and $b$ are given by $a = 7 + 6\mathrm{i}$ and $b = 1 - 3\mathrm{i}$. Showing clearly how you obtain your answers, find

**(i)** $|a - 2b|$ and $\arg(a - 2b)$, **[4]**

**(ii)** $\dfrac{b}{a}$, giving your answer in the form $x + \mathrm{i}y$. **[3]**

**5** **(a)** Write down the matrix that represents a reflection in the line $y = x$. **[2]**

**(b)** Describe fully the geometrical transformation represented by each of the following matrices:

**(i)** $\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, **[2]**

**(ii)** $\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$. **[2]**

**6** **(i)** Sketch on a single Argand diagram the loci given by

**(a)** $|z - 3 + 4\mathrm{i}| = 5$, **[2]**

**(b)** $|z| = |z - 6|$. **[2]**

**(ii)** Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 + 4\mathrm{i}| \leqslant 5 \quad \text{and} \quad |z| \geqslant |z - 6|. \qquad \textbf{[2]}$$

**7** The quadratic equation $x^2 + 2kx + k = 0$, where $k$ is a non-zero constant, has roots $\alpha$ and $\beta$. Find a quadratic equation with roots $\dfrac{\alpha + \beta}{\alpha}$ and $\dfrac{\alpha + \beta}{\beta}$. **[7]**

**8** **(i)** Show that $\dfrac{1}{\sqrt{r+2}+\sqrt{r}} \equiv \dfrac{\sqrt{r+2}-\sqrt{r}}{2}$. **[2]**

**(ii)** Hence find an expression, in terms of $n$, for

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r+2}+\sqrt{r}}.$$ **[6]**

**(iii)** State, giving a brief reason, whether the series $\displaystyle\sum_{r=1}^{\infty} \frac{1}{\sqrt{r+2}+\sqrt{r}}$ converges. **[1]**

**9** The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & a & -1 \\ 0 & a & 2 \\ 1 & 2 & 1 \end{pmatrix}$.

**(i)** Find, in terms of $a$, the determinant of $\mathbf{A}$. **[3]**

**(ii)** Three simultaneous equations are shown below.

$$\begin{aligned} ax + ay - z &= -1 \\ ay + 2z &= 2a \\ x + 2y + z &= 1 \end{aligned}$$

For each of the following values of $a$, determine whether the equations are consistent or inconsistent. If the equations are consistent, determine whether or not there is a unique solution.

**(a)** $a = 0$

**(b)** $a = 1$

**(c)** $a = 2$

**[6]**

**10** The complex number $z$, where $0 < \arg z < \frac{1}{2}\pi$, is such that $z^2 = 3 + 4\mathrm{i}$.

**(i)** Use an algebraic method to find $z$. **[5]**

**(ii)** Show that $z^3 = 2 + 11\mathrm{i}$. **[1]**

The complex number $w$ is the root of the equation

$$w^6 - 4w^3 + 125 = 0$$

for which $-\frac{1}{2}\pi < \arg w < 0$.

**(iii)** Find $w$. **[5]**

**1**

B1 Establish result true for $n = 1$ or $n = 2$
M1 Add next term to given sum formula
M1 Attempt to factorise or expand and simplify to correct expression
A1 Correct expression obtained
A1 **5** Specific statement of induction conclusion

**5**

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**2** **(i)** $(-7)$

M1 Obtain a single value
A1 **2** Obtain correct answer as a matrix

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**(ii)** $\mathrm{BA} = \begin{pmatrix} 5 & -20 \\ 3 & -12 \end{pmatrix}$

$\begin{pmatrix} -7 & -20 \\ 11 & -20 \end{pmatrix}$

M1 Obtain a $2 \times 2$ matrix
A1 All elements correct
B1 4**C** seen or implied by correct answer
B1ft **4** Obtain correct answer, ft for a slip in **BA**

**6**

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**3** Either

$\frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$

$\frac{1}{3}n(2n-1)(2n+1)$

M1 Express as a sum of 3 terms
M1 Use standard sum results
A1 Correct unsimplified answer
M1 Attempt to factorise
A1 Obtain at least factor of $n$ and a quadratic
A1 **6** Obtain correct answer a.e.f.

***Or***

$\sum_{r=1}^{2n} r^2 - 4\sum_{r=1}^{n} r^2$

$\frac{1}{6} \times 2n(2n+1)(4n+1) - 4 \times \frac{1}{6}n(n+1)(2n+1)$

$\frac{1}{3}n(2n-1)(2n+1)$

M1 Express as difference of 2 $\sum r^2$ series
M1 Use standard result
A1 Correct unsimplified answer
M1 Attempt to factorise
A1 Obtain at least factor of $n$
A1 Obtain correct answer

**6**

| | | | | |
|---|---|---|---|---|
| **4** | **(i)** | 5 + 12i<br>13<br>$67.4^o$ or 1.18 | B1B1<br>B1ft<br>B1ft **4** | Correct real and imaginary parts<br>Correct modulus<br>Correct argument |
| | **(ii)** | $-\frac{11}{85}-\frac{27}{85}$i | M1<br>A1<br>A1 **3** | Multiply by conjugate<br>Obtain correct numerator<br>Obtain correct denominator |
| | | | **7** | |
| **5** | **(a)** | $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ | B1B1**2** | Each column correct<br>**SC B2 use correct matrix from MF1**<br>**Can be trig form** |
| | **(b)** | **(i)** | B1B1**2** | Stretch, in $x$-direction sf 5 |
| | | **(ii)** | B1B1**2** | Rotation, $60^o$ clockwise |
| | | | **6** | |
| **6** | **(i)** | **(a)** | B1B1**2** | Circle centre (3, –4 ), through origin |
| | | **(b)** | B1B1**2** | Vertical line, clearly $x = 3$ |
| | **(ii)** | | B1ft<br>B1ft **2** | Inside their circle<br>And to right of their line, if vertical |
| | | | **6** | |

**7** *Either*

$\alpha + \beta = -2k \quad \alpha\beta = k$

B1B1 State or use correct results

M1 Attempt to find sum of new roots

A1 Obtain $4k$

M1 Attempt to find product of new roots

A1 Obtain $4k$

B1ft **7** Correct quadratic equation a.e.f.

$y^2 - 4ky + 4k = 0$

***Or***

$\alpha + \beta = -2k$

$\frac{-2k}{\alpha}$

$y = \frac{-2k}{x}$

B1 State or use correct result

B1 State or imply form of new roots

B1 State correct substitution

M1 Rearrange and substitute for $x$

A1 Correct unsimplified equation

M1 Attempt to clear fractions

A1 Correct quadratic equation a.e.f.

$y^2 - 4ky + 4k = 0$

***Or***

$-k \pm \sqrt{k^2 - k}$

B1 Find roots of original equation

$\frac{\alpha + \beta}{\alpha} = \frac{2k}{k + \sqrt{k^2 - k}}, \frac{\alpha + \beta}{\beta} = \frac{2k}{k - \sqrt{k^2 - k}}$

B1 Express both new roots in terms of $k$

M1 Attempt to find sum of new roots

A1 Obtain $4k$

M1 Attempt to find product of new roots

A1 Obtain $4k$

$y^2 - 4ky + 4k = 0$

B1ft Correct quadratic equation a.e.f.

7

| | | | | | |
|---|---|---|---|---|---|
| **8** | **(i)** | | M1 | | Attempt to rationalise denominator or cross multiply |
| | | | A1 | **2** | Obtain **given** answer correctly |
| | **(ii)** | | M1 | | Express terms as differences using (i) |
| | | | M1 | | Attempt this for at least 1st three terms |
| | | | A1 | | 1st three terms all correct |
| | | | A1 | | Last two terms all correct |
| | | | M1 | | Show pairs cancelling |
| | | $\frac{1}{2}(\sqrt{n+2}+\sqrt{n+1}-\sqrt{2}-1)$ | A1 | **6** | Obtain correct answer, in terms of $n$ |
| | **(iii)** | | B1 | **1** | Sensible statement for divergence |
| | | | | **9** | |
| **9** | **(i)** | | M1 | | Show correct expansion process for 3 x 3 |
| | | | M1 | | Correct evaluation of any 2 x 2 |
| | | det **A** = $a^2 - a$ | A1 | **3** | Obtain correct answer |
| | **(ii)** | **(a)** | M1 | | Find a pair of inconsistent equations |
| | | | A1 | | State inconsistent or no solutions |
| | | **(b)** | M1 | | Find a repeated equation |
| | | | A1 | | State non unique solutions |
| | | **(c)** | B1 | | State that det **A** is non-zero or find correct solution |
| | | | B1 | **6** | State unique solution |
| | | | | | **SC if detA incorrect, can score 2 marks for correct deduction of a unique solution, but only once** |
| | | | | **9** | |
| **10** | **(i)** | | M1 | | Attempt to equate real and imaginary parts |
| | | $x^2 - y^2 = 3 \quad xy = 2$ | A1 | | Obtain both results |
| | | | M1 | | Eliminate to obtain quadratic in $x^2$ or $y^2$ |
| | | | M1 | | Solve to obtain $x$ or $y$ value |
| | | $z = 2 + \mathrm{i}$ | A1 | **5** | Obtain correct answer as a complex no. |
| | **(ii)** | | B1 | **1** | Obtain **given** answer correctly |
| | **(iii)** | | M1 | | Attempt to solve quadratic equation |
| | | $w^3 = 2 \pm 11\,\mathrm{i}$ | A1 | | Obtain correct answers |
| | | | M1 | | Choose negative sign |
| | | | M1 | | Relate required value to conjugate of (i) |
| | | $w = 2 - \mathrm{i}$ | A1 | **5** | Obtain correct answer |
| | | | | **11** | |